Mathematics Subject Matter Requirements

Part I: Content Domains for Subject Matter Understanding and Skill in Mathematics

Domain 1. Number and Quantity

Candidates demonstrate an understanding of number theory and a command of number sense as outlined in the California Common Core Content Standards for Mathematics (Grade 6, Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of number systems and their underlying structures. They prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning and verify conjectures with proofs.

1.1 The Real and Complex Number Systems

a. Demonstrate knowledge of the properties of the real number system and of its subsets
b. Perform operations and recognize equivalent expressions using various representations of real numbers (e.g., fractions, decimals, exponents)
c. Solve real-world and mathematical problems using numerical and algebraic expressions and equations
d. Apply proportional relationships to model and solve real-world and mathematical problems
e. Reason quantitatively and use units to solve problems (i.e., dimensional analysis)
f. Perform operations on complex numbers and represent complex numbers and their operations on the complex plane

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: The Number System, Grade 7 [7.NS]; The Real Number System, Grade 8; Quantities, High School [N-Q]; Expressions and Equations, Grade 7 [7.EE]; Ratios and Proportional Relationships, Grade 7 [7.RP]; The Real Number System, High School [N-RN]; The Complex Number System, High School [N-CN])

1.2 Number Theory

a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)
b. Use the principle of mathematical induction to prove results in number theory
c. Apply the Euclidean Algorithm
d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple; show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime; prove that the square root of any number, not a perfect square number, is irrational)

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: The Number System, Grade 6 [6.NS])

Domain 2. Algebra

Candidates demonstrate an understanding of the foundations of algebra as outlined in the California Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of algebra and its underlying structures. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.

2.1 Algebraic Structures

a. Demonstrate knowledge of why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings)
b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$)
c. Demonstrate knowledge that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field
d. Identify and translate between equivalent forms of algebraic expressions and equations using a variety of techniques (e.g., factoring, applying properties of operations)
e. Justify the steps in manipulating algebraic expressions and solving algebraic equations and inequalities
f. Represent situations and solve problems using algebraic equations and inequalities

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: The Real Number System, High School [N-RN]; The Complex Number System, High School [N-CN]; Seeing Structure in Expressions, High School [A-SSE]; Reasoning with Equations and Inequalities, High School [A-REI]; Creating Equations, High School [A-CED])

2.2 Polynomial Equations and Inequalities

a. Analyze and solve polynomial equations with real coefficients using:
   - the Fundamental Theorem of Algebra
   - the Rational Root Theorem for polynomials with integer coefficients
   - the Conjugate Root Theorem for polynomial equations with real coefficients
   - the Binomial Theorem
b. Prove and use the Factor Theorem and the quadratic formula for real and complex quadratic polynomials
c. Solve polynomial inequalities

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Reasoning with Equations and Inequalities, High School [A-REI]; Arithmetic with Polynomials and Rational Expressions, High School [A-APR]; Linear, Quadratic, and Exponential Models, High School [F-LE])

2.3 Functions

a. Analyze general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) and apply arithmetic operations on functions
b. Analyze properties of linear functions (e.g., slope, intercepts) using a variety of representations
c. Demonstrate knowledge of why graphs of linear inequalities are half planes and be able to apply this fact
d. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)
e. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)
f. Model and solve problems using nonlinear functions

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Interpreting Functions, High School [F-IF]; Building Functions, High School [F-BF]; Linear, Quadratic, and Exponential Models, High School [F-LE])

2.4 Linear Algebra

a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples
b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product)
c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations)
d. Analyze the properties of proportional relationships, lines, linear equations, and their graphs, and the connections between them
e. Model and solve problems using linear equations, pairs of simultaneous linear equations, and their graphs

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Vector and Matrix Quantities, High School [N-VM]; Expressions and Equations, Grade 8; Linear, Quadratic, and Exponential Models, High School [F-LE]; Ratios and Proportional Relationships, Grade 7 [7.RP])

Domain 3. Geometry

Candidates demonstrate an understanding of the foundations of geometry as outlined in the California Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of geometry and its underlying structures. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry.

3.1 Plane Euclidean Geometry

a. Apply the Parallel Postulate and its implications and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)
b. Demonstrate knowledge of complementary, supplementary, and vertical angles
c. Prove theorems, justify steps, and solve problems involving similarity and congruence
d. Apply and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, triangle inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)
e. Apply and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
f. Identify and justify the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular polygons with 3, 4, 5, 6, and 8 sides)

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Geometry, Grade 7 [7.G]; Geometry, Grade 8; Congruence, High School [G-CO]; Similarity, Right Triangles, and Trigonometry, High School [G-SRT]; Circles, High School [G-C]; Geometric Measurement and Dimension, High School [G-GMD])

3.2 Coordinate Geometry

a. Use techniques in coordinate geometry to prove geometric theorems
b. Model and solve mathematical and real-world problems by applying geometric concepts to two-dimensional figures
c. Translate between the geometric description and the equation for a conic section
d. Translate between rectangular and polar coordinates and apply polar coordinates and vectors in the plane

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Geometry, Grade 8; Expressing Geometric Properties with Equations, High School [G-GPE]; Geometric Measurement and Dimension, High School [G-GMD]; Modeling with Geometry, High School [G-MG]; Polar Coordinates and Curves, High School)

3.3 Three-Dimensional Geometry

a. Demonstrate knowledge of the relationships between lines and planes in three dimensions (e.g., parallel, perpendicular, skew, coplanar lines)
b. Apply and justify properties of three-dimensional objects (e.g., the volume and surface area formulas for prisms, pyramids, cones, cylinders, spheres)
c. Model and solve mathematical and real-world problems by applying geometric concepts to three-dimensional figures

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Congruence, High School [G-CO]; Similarity, Right Triangles, and Trigonometry, High School [G-SRT]; Geometric Measurement and Dimension, High School [G-GMD]; Modeling with Geometry, High School [G-MG])

3.4 Transformational Geometry

a. Demonstrate knowledge of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection), including their basic properties in relation to congruence
b. Demonstrate knowledge of dilations (e.g., similarity transformations or change in scale factor), including their basic properties in relation to similarity, volume, and area

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Geometry, Grade 8; Congruence, High School [G-CO])

Domain 4. Probability and Statistics

Candidates demonstrate an understanding of statistics and probability distributions as outlined in the California Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of probability and statistics and their underlying structures. They solve problems and make inferences using statistics and probability distributions.

4.1 Probability

a. Prove and apply basic principles of permutations and combinations
b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles, sample space)
c. Use and explain the concepts of conditional probability and independence
d. Compute and interpret the probability of an outcome, including the probabilities of compound events in a uniform probability model
e. Use normal, binomial, and exponential distributions to solve and interpret probability problems
f. Calculate expected values and use them to solve problems and evaluate outcomes of decisions

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Statistics and Probability, Grade 7 [7.SP]; Conditional Probability and the Rules of Probability, High School [S-CP]; Using Probability to Make Decisions, High School [S-MD])

4.2 Statistics

a. Compute and interpret the mean and median of both discrete and continuous distributions
b. Compute and interpret quartiles, range, interquartile range, and standard deviation of both discrete and continuous distributions
c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results
d. Apply the method of least squares to linear regression
e. Apply the chi-square test
f. Interpret scatter plots for bivariate data to investigate patterns of association between two quantities (e.g., correlation), including the use of linear models
g. Interpret data on a single count or measurement variable presented in a variety of formats (e.g., dot plots, histograms, box plots)

h. Demonstrate knowledge of P-values and hypothesis testing
i. Demonstrate knowledge of confidence intervals

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Statistics and Probability, Grade 8; Interpreting Categorical and Quantitative Data, High School [S-ID])

Domain 5. Calculus

Candidates demonstrate an understanding of trigonometry and calculus as outlined in the California Common Core Content Standards for Mathematics (High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of trigonometry and calculus and their underlying structures. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.

5.1 Trigonometry

a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity $\sin^2 x + \cos^2 x = 1$ and that this identity leads to $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$
b. Prove and apply the sine, cosine, and tangent sum formulas for all real values
c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems, using the unit circle)
d. Apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan)
e. Apply polar representations of complex numbers (e.g., DeMoivre's Theorem)
f. Model periodic phenomena with periodic functions
g. Recognize equivalent identities, including applications of the half-angle and double-angle formulas for sines and cosines

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Trigonometric Functions, High School [F-TF])

5.2 Limits and Continuity

a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit
b. Show that a polynomial function is continuous at a point
c. Apply the intermediate value theorem, using the geometric implications of continuity

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Calculus Standards, High School)

5.3 Derivatives and Applications

a. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative
b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)

c. Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule
d. Use the derivative to solve rectilinear motion, related rate, and optimization problems
e. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity)
f. Solve separable first-order differential equations and apply them to growth and decay problems

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Calculus Standards, High School)

5.4 Integrals and Applications

a. Derive definite integrals of standard algebraic functions using the formal definition of integral
b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums)
c. Prove the fundamental theorem of calculus, and use it to interpret definite integrals as antiderivatives
d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Calculus Standards, High School)

5.5 Sequences and Series

a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)
b. Determine convergence of a given sequence or series using standard techniques (e.g., ratio, comparison, integral tests)
c. Calculate Taylor series and Taylor polynomials of basic functions

(California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Seeing Structure in Expressions, High School [A-SSE]; Calculus Standards, High School)

Part II: Subject Matter Skills and Abilities Applicable to the Content Domains in Mathematics

Candidates for Single Subject Teaching Credentials in mathematics use inductive and deductive reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason both abstractly and quantitatively, they use counterexamples, construct proofs using contradictions, construct viable arguments, and critique the reasoning of others. They create multiple representations of the same concept. They know the interconnections among mathematical ideas, use appropriate tools strategically, and apply techniques and concepts from different domains and sub-domains to model the same problem. They explain mathematical interconnections with other disciplines. They are able to communicate their mathematical thinking clearly and coherently to others, orally, graphically, and in writing. They attend to precision, including the use of precise language and symbols.

Candidates make sense of routine and complex problems, solving them by selecting from a variety of strategies. They look for and make use of structure while demonstrating persistence and reflection in their approaches. They analyze problems through pattern recognition, look for and express regularity in repeated reasoning, and use analogies. They formulate and prove conjectures, and test conclusions for reasonableness and accuracy. They use counterexamples to disprove conjectures.

Candidates select and use different representational systems (e.g., coordinates, graphs). They understand the usefulness of transformations and symmetry to help analyze and simplify problems. They model with mathematics to analyze mathematical structures in real contexts. They use spatial reasoning to model and solve problems that cross disciplines.

(California Common Core Content Standards for Mathematics [Grade 7, Grade 8, and High School], including Standards for Mathematical Practice 1–8)

Note: These subject matter requirements have been reorganized to reflect the test structure of CSET: Mathematics and so differ in order from the Single Subject Mathematics Subject Matter Requirements as adopted by the Commission on Teacher Credentialing.